Lista n°8

Exercício 1

a. Calcular

$$\Delta_2 = \begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix} \quad \text{e} \quad \Delta_3 = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix}$$

b. Calcular Δ_n, o determinante da matriz quadrada $n*n$ com todos os elementos da diagonale iguais a a e todos os outros elementos iguais a 1.

Exercício 2

Calcular os determinantes seguintes :

$$\begin{vmatrix} 246 & 427 & 327 \\ 1014 & 543 & 443 \\ -342 & 721 & 621 \end{vmatrix}, \quad \begin{vmatrix} 13547 & 13647 \\ 28423 & 28523 \end{vmatrix}.$$

Exercício 3

Calcular os determinantes seguintes :

$$\begin{vmatrix} 1 & 1 & 1 \\ \cos a & \cos b & \cos c \\ \sin a & \sin b & \sin c \end{vmatrix}, \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}, \begin{vmatrix} a+b & ab & a^2+b^2 \\ b+c & bc & b^2+c^2 \\ c+a & ca & c^2+a^2 \end{vmatrix}.$$

Exercício 4

Seja $\omega = \cos(2\pi/3) + i\sin(2\pi/3)$.

a. Provar que $1 + \omega + \omega^2 = 0$.

b. Calcular

$$D = \begin{vmatrix} \omega & \omega \\ -1 & \omega \end{vmatrix} \quad \text{et} \quad D' = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$

Exercício 5

Provar que para a, b e c números reais, temos :

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3 .$$

Exercício 6

Provar, sem calcular, que os determinantes seguintes são nulos :

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix} \quad \text{e} \quad D_2 = \begin{vmatrix} a^2 & (a+1)^2 & (a+2)^2 & (a+3)^2 \\ b^2 & (b+1)^2 & (b+2)^2 & (b+3)^2 \\ c^2 & (c+1)^2 & (c+2)^2 & (c+3)^2 \\ d^2 & (d+1)^2 & (d+2)^2 & (d+3)^2 \end{vmatrix}$$

Exercício 7

Seja $a, b, c \in \mathbb{R}$. Calcular os determinantes seguintes :

$$D_1 = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \end{vmatrix}, \; D_2 = \begin{vmatrix} a+b+c & b & b & b \\ c & a+b+c & b & b \\ c & c & a+b+c & b \\ c & c & c & a+b+c \end{vmatrix}, \; D_3 = \begin{vmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ a & 0 & a & 0 & 3 \\ b & a & 0 & a & 0 \\ 0 & b & 0 & 0 & a \end{vmatrix}$$

Exercício 8

Sejam $a_1, \cdots, a_n \in \mathbb{R}$. Calcular os determinantes $n \times n$ seguintes :

$$D_1 = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ a_1 & a_2 & \cdots & a_n \\ a_1^2 & a_2^2 & \cdots & a_n^2 \\ \vdots & & & \vdots \\ a_1^{n-1} & a_2^{n-1} & \cdots & a_n^{n-1} \end{vmatrix}, \; D_2 = \begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_n \\ a_2 & a_2 & a_3 & \cdots & a_n \\ a_3 & a_3 & a_3 & \cdots & a_n \\ \vdots & & & & \vdots \\ a_n & a_n & a_n & \cdots & a_n \end{vmatrix}$$

Exercício 9

Calcular os determinantes seguintes :

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ \cos x & \cos y & \cos z & \cos t \\ \cos 2x & \cos 2y & \cos 2z & \cos 2t \\ \cos 3x & \cos 3y & \cos 3z & \cos 3t \end{vmatrix} \quad \begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ -1 & 0 & 3 & & n \\ -1 & -2 & 0 & & n \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & -2 & -3 & \cdots & 0 \end{vmatrix} \quad \begin{vmatrix} 0 & 1 & 2 & \cdots & n-1 \\ 1 & 0 & 1 & \ddots & \cdots \\ 2 & 1 & \ddots & \ddots & 2 \\ \vdots & \ddots & \ddots & 0 & 1 \\ n-1 & \cdots & 2 & 1 & 0 \end{vmatrix}$$

Exercício 10

Calcular o determinante seguinte :

$$\Delta = \begin{vmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{vmatrix}$$

Exercício 11

Calcular os determinantes seguintes :

a. $$\begin{vmatrix} 1 & 0 & \dots & \dots & 0 & 1 \\ 0 & 0 & & & 0 & 0 \\ \vdots & & & & & \vdots \\ \vdots & & & & & \vdots \\ 0 & 0 & & & 0 & 0 \\ 1 & 0 & \dots & \dots & 0 & 1 \end{vmatrix}$$

b. $\begin{vmatrix} 1 & \dots & & \dots & 1 \\ \vdots & 0 & 1 & \dots & 1 \\ & 1 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 1 & 1 & \dots & 1 & 0 \end{vmatrix}$ e $\begin{vmatrix} 0 & 1 & \dots & \dots & 1 \\ 1 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 1 \\ 1 & \dots & \dots & 1 & 0 \end{vmatrix}$ $(n \geqslant 2)$

Exercício 12

Calcular os determinantes seguintes :

$$\begin{pmatrix} 7 & 11 \\ -8 & 4 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 6 \\ 3 & 4 & 15 \\ 5 & 6 & 21 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & 5 \\ 4 & 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 1 & 3 \\ 2 & 1 & 0 & 6 \\ 1 & 1 & 1 & 7 \end{pmatrix}$$

Exercício 13

Calcular os determinantes seguintes :

$$\begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 2 & 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 10 & 0 & -5 & 15 \\ -2 & 7 & 3 & 0 \\ 8 & 14 & 0 & 2 \\ 0 & -21 & 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} a & a & b & 0 \\ a & a & 0 & b \\ c & 0 & a & a \\ 0 & c & a & a \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 \\ a & 0 & a & 0 & 3 \\ b & a & 0 & a & 0 \\ 0 & b & 0 & 0 & a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & -4 & 3 & 0 & 0 \\ -3 & 0 & 0 & -3 & -2 \\ 0 & 1 & 7 & 0 & 0 \\ 4 & 0 & 0 & 7 & 1 \end{pmatrix}$$

Exercício 14

Calcular os determinantes seguintes :

$$\begin{vmatrix} a_1 & a_2 & \cdots & a_n \\ a_1 & a_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & a_2 \\ a_1 & \cdots & a_1 & a_1 \end{vmatrix} \qquad \begin{vmatrix} 1 & & & 1 \\ 1 & 1 & (0) & \\ & \ddots & \ddots & \\ (0) & & 1 & 1 \end{vmatrix} \qquad \begin{vmatrix} a+b & a & \cdots & a \\ a & a+b & \ddots & \vdots \\ \vdots & \ddots & \ddots & a \\ a & \cdots & a & a+b \end{vmatrix}$$