Lista de exercícios n°10

Exercício 1

Calcular as integrais seguintes :

a. $\int_0^1 (x^6 - 2x^3 + 3x - 8)dx$

b. $\int_0^\pi \cos x dx$

c. $\int_{-1}^2 (6x + \text{sen}x - 1)(2 - x + 3x^2 - \cos x)^4 dx$

d. $\int_0^1 \frac{2x+1}{(x^2+x+1)^3} dx$

e. $\int_1^2 \frac{1+2x}{1+(x^2+x+1)^2} dx$

f. $\int_0^1 \frac{2x+1}{(x^2+x+1)} dx$

g. $\int_0^{\pi/4} \frac{\text{sen}x}{\cos x} dx$

h. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\text{sen}x} dx$

i. $\int_0^1 (10x^4 - 2)e^{1-2x+2x^5} dx$

j. $\int_1^2 \frac{2+\cos x}{\sqrt{2x+\text{sen}x+1}} dx$

Exercício 2

Calcular as integrais seguintes :

$a)\ \displaystyle\int_1^e \ln(x)dx \qquad b)\ \int_0^1 xe^{-x} \qquad c)\ \int_0^{\pi/2} x\text{sen}(2x)dx$

$d)\ \displaystyle\int_0^{\frac{\pi}{2}} x^2\cos(3x)dx \qquad e)\ \int_1^2 \frac{-1}{\sqrt{4x-x^2}}dx \qquad f)\ \int_{1/2}^2 \frac{1}{x\sqrt{1-\ln^2 x}}dx$ sur]1/e,e[

$g)\ \displaystyle\int_0^1 \frac{\exp(2x)}{\sqrt{\exp(2x)+1}}dx$

Exercício 3

Calcular as integrais seguintes :

$a)\ \displaystyle\int_1^2 \frac{1}{x\ln x}dx \qquad b)\ \int_0^1 \frac{x}{\sqrt{x+1}}dx \qquad c)\ \int_1^2 \text{sen}(\ln(x))dx$

$d)\ \displaystyle\int_0^\pi \cos x \exp x dx \qquad e)\ \int_0^1 \frac{1}{3+\exp(-x)}dx$

Exercício 4

Calcular as integrais seguintes :

$$\int_0^{\pi/2} \frac{\text{sen}x}{\text{sen}x + \cos x}dx \qquad \text{e} \qquad \int_0^{\pi/2} \frac{\cos x}{\cos x + \text{sen}x}dx$$

Exercício 5

Calcular as integrais seguintes usando substituição :

$a)\ \displaystyle\int_0^1 \frac{1}{\exp(x)+1}dx \qquad b)\ \int_0^1 \frac{1}{(1+x^2)^2}dx \quad (x = \tan(u))$

$c)\ \displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos(x)}{6-5\text{sen}(x)+\text{sen}^2(x)}dx \quad (u = \text{sen}x) \qquad d)\ \int_0^{\pi/2} \frac{1}{\cos(x)+\text{sen}(x)}dx \quad (u = \tan\frac{x}{2})$

$e)\ \displaystyle\int_0^a \sqrt{a^2-x^2}dx.$

Exercício 6

Usando a substituição $x = \tan(u)$, calcular :

$$\int_0^1 \text{arcsen}(\frac{2x}{1+x^2})dx.$$

Dica : voce pode usar que $\text{sen}(2u) = \frac{2\tan(u)}{1+\tan^2(u)}$

Exercício 7

Calcular as integrais seguintes :

$$a) \int_0^1 \frac{x^2-5x+9}{x^2-5x+6}dx \quad b) \int_0^1 \frac{1}{t^2+5t+6}dt \quad c) \int_0^1 \frac{1}{(t^2+1)(t+1)}dt$$

$$d) \int_0^1 \frac{x}{x^2+2x+10}dx \quad e) \int_{-2}^{-1} \frac{5x+2}{x^3-5x^2+4x}dx \quad f) \int_1^2 \frac{1}{x(x+1)^2}dx$$

$$g) \int_0^1 \frac{x}{(x+1)^2(x^2+1)}dx \quad h) \int_0^1 \frac{3x^2-x+11}{(4+x^2)^2}$$

Exercício 8

$$a) \int_0^\pi \cos^4(x)dx \quad b) \int_0^\pi \cos^{2012} x\text{sen}x dx \quad c) \int_0^\pi \text{sen}^8 x \cos^3 x dx$$

$$d) \int_0^{\pi/4} \frac{1}{\cos x}dx$$

Exercício 9

1. Calcular

$$\int_1^e \frac{x}{(1+x^2)^2}dx$$

2. Decompor em frações parciais a função f definida por

$$f(x) = \frac{2x^2-3x+2}{(1+x^2)^2}.$$

3. Calcular $\int_1^e f(x)dx$.

4. Usando substituição e 3) calcular :

$$\int_0^1 \frac{(2e^{2x}-3e^x+2)e^x}{(1+e^{2x})^2}dx$$

Exercício 10

Achar uma primitiva das funções seguintes :

a. $f(x) = (e^x + e^{-2x})^2$;

b. $g(x) = \text{sen} x\sqrt{1+\cos x}$;

c. $h(x) = \frac{1}{x^2}\arctan x$.

Exercício 11

Soit f une fonction continue sur $[a, b]$. Montrer qu'il existe $c \in]a, b[$ tel que

$$\int_a^b f(t)\mathrm{d}t = f(c).$$